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Students' Misconceptions Regarding the Principle of Superposition and The Optimum Power Transfer in Electric Circuits

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Abstract

The article describes a problem-based approach (applied in lectures at the Dept. of Electrical & Electronic Engineering Educators, School of Pedagogical & Technological Education (ASPETE), Athens, Greece) aiming at handling students' misconceptions regarding the principle of superposition and the optimum power transfer in electrical circuits. This approach was applied following observations that students would apply superposition without really checking its applicability to the problem under study and would associate the optimization of power transfer to the maximization of voltage load. Though the principle of superposition can be proved through basic properties of linear equations and the theorem of the optimum power transfer can be handled as a typical maximization problem, to help the students develop confidence in the relevant topics, the lectures started with the solution of specifically selected problems (in which the students were actively involved) before proceeding to mathematical proofs.

Introduction

It has been observed that regarding electrical circuits, several students develop misconceptions related to, among others, circuit theorems such as the principle of superposition and the optimum power transfer. For example, in a lecture on simple transistor amplifier circuits (for 3rd semester students) the great majority of students did not realize that the use of different DC and AC equivalent circuits had been an outcome of the principle of superposition which, in turn, pre-assumed that transistors should be considered as linear. In the same questionnaire, about 80% of the students chose $R_L >> R_S$ (load resistor much larger than the source resistor) as the necessary condition for optimum power transfer from a source to the load justifying their choice by arguing that this condition should be the same to the one that maximizes the load voltage.

Following the above observations and with the aim of preventing students from developing such misconceptions, the presentation of the principle of superposition and the optimum power transfer (within the framework of the 1st semester "Electric Circuits" course) gave emphasis to illustrating the basic principles, pre-assumptions and applicability of the two theorems through specifically chosen problems. In this context, the linearity requirement for superposition was made evident by means of a counter example in which one of the resistors was nonlinear while the handling of dependent sources was referred to through a simple example that the students had to solve based on instructor's hints. Similarly, for the optimum power transfer, a purely ohmic Thevenin equivalent circuit was examined followed by a brief reference to a Thevenin equivalent with complex impedances.

Methodology

The Principle of Superposition

The principle of superposition (under the necessary condition that the circuit elements are linear) can be easily proved by using basic properties of linear equations and systems (algebraic or differential) [1]. In order, however, the students to appreciate the pre-assumptions, applicability, and limitations of superposition (as applied to electric circuits), the following two problems were given to them:

a. A single loop ohmic circuit (like the one shown in figure. 1) with two DC voltage sources and two linear resistors. More specifically, it is $v_1 = 10V$, $v_2 = 5V$, $R_1 = 2\Omega$ and $R_2 = 3\Omega$.

b. A single loop ohmic circuit (like the one shown in figure. 1) with two DC voltage sources and a non-linear resistor with a value proportional to the circuit's current i. More specifically, it is $v_1 = 10V$, $v_2 = 4V$, $R_1 = i(\Omega)$ (that is, $R_1 = a.i$ with $a = 1\Omega/A$) and $R_2 = 3\Omega$.

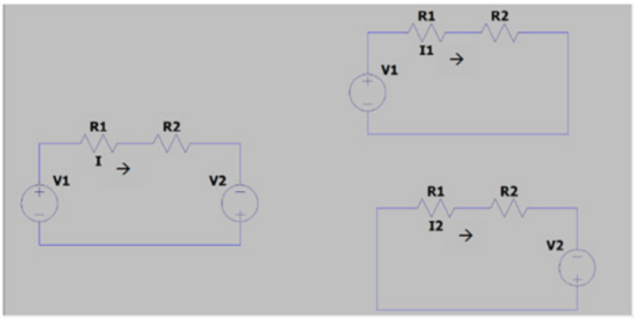

Figure 1: A single-loop ohmic circuit with two DC voltage sources. In problem (1), $R_1 = 2\Omega$ (linear) while in problem (2), $R_1 = i(\Omega)$ (that is, non-linear) where i (in A) is the circuit's current.

The students were asked to solve the given problems, first by applying Kirchhoff's voltage law in the original circuit on the left and then by employing the principle of superposition in the two sub-circuits on the right.

Regarding problem (a) it was easy to show that i = 3A, $i_1 = 2A$ and $i_2 = 1A$ which means that $i_1+i_2 = i$ as anticipated by the principle of superposition. However, in problem (b), i = 2.53A, $i_1 = 2A$ and $i_2 = 1A$, hence $i_1+i_2 \neq i$. This means that, in this circuit, superposition would give false results owing to the fact that the circuit includes a non-linear element (the resistor $R_1 = i\ \Omega$) which renders the principle of superposition not applicable.

Finally, the students were asked to calculate power dissipation of R_1 in problem (a) (where the principle of superposition was applicable) by, first, using the results for the original circuit (on the left) and then using the results for the two sub-circuits on the right. In the original circuit, it was $P_{R1} = i^2R_1 = 18W$ a different value than that obtained through the superimposition of the two sub-circuits on the right which was $i_1^2R_1+i_2^2R_1 = 10W$. This means that superposition would give false results owing to the fact that the dissipated power is a non-linear quantity or, in algebraic terms, to the fact that while $i = i_1+i_2$, it is $i^2 = (i_1+i_2)^2 \neq i_1^2+i_2^2$.

Through the problems above, it was demonstrated that the principle of superposition

a. applies to circuits that only include linear elements (in this case, resistors for which the current is proportional to the voltage),

b. can only be used for the calculation of linear quantities (such as voltages and currents) and not for the evaluation of non-linear ones (such as the power dissipation).

Following the above calculations, the students were considered more ready to appreciate a straightforward mathematical proof based on the well-known property of linear algebraic equations that the sum of two solutions (in general, a linear combination of those solutions) is also a solution of the given equation. The proof aimed at helping students get familiar with the mathematics of the superposition in addition to the example-based presentation.

A reference was also made to circuits that, apart from linear resistors, include other circuit elements such as linear inductors and/or capacitors. Those elements are described by equations such as

$v(t) = L\, di/dt$

$i(t) = C\, dv/dt$

that involve the derivative of either the current (inductors) or the voltage (capacitors) and, as such, do not lend themselves for the use of straightforward examples as in ohmic circuits. Thus, to demonstrate the applicability of the principle of superposition, the circuit of figure. 2 was analyzed. By applying Kirchhoff's voltage law, it was easy to show that with v_1 and v_2 acting alone

$$v_1 = v_{R1} + v_{L1} \Rightarrow v_1 = Ri_1 + L\frac{di_1}{dt} \Rightarrow \frac{di_1}{dt} + \frac{R}{L}i_1 = \frac{v_1}{L}$$

$$v_2 = v_{R2} + v_{L2} \Rightarrow v_2 = Ri_2 + L\frac{di_2}{dt} \Rightarrow \frac{di_2}{dt} + \frac{R}{L}i_2 = \frac{v_2}{L}$$

that, if superimposed (added in parts) result in the equation

$$\frac{d(i_1+i_2)}{dt} + \frac{R}{L}(i_1+i_2) = \frac{(v_1+v_2)}{L}$$

which (since $i_1 + i_2 = i$, the overall current of the circuit) results in the equation

$$\frac{di}{dt} + \frac{R}{L}i = \frac{(v_1+v_2)}{L}$$

that regards the original circuit (with the two voltage sources). Through the above calculations, the students would better appreciate the fact that the principle of superposition is applicable in any system (electrical or non-electrical) described by linear equations whether algebraic or differential.

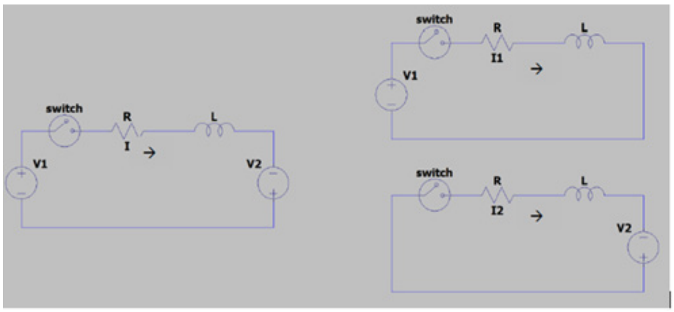

Figure 2: A single-loop circuit with two DC voltage sources (fig. 2) a resistor and an inductor.

Another misunderstanding regards circuits that include dependent sources. Again, by solving a simple problem, the student saw that the dependent source has to be included in all the sub-circuits obtained by decomposing the original one.

A final comment (aiming at demonstrating the usefulness of superposition and preparing students for subsequent electrical and electronic engineering topics) regarded its application to electronic circuits such as transistor amplifiers of the common-emitter or the common-collector type. Though transistors are typically non-linear elements, they can be considered as approximately linear when the input signals are sufficiently small, in which case the application of superposition decomposes the original circuit into a purely DC and a purely AC circuit that are much easier to analyze [2].

Optimum power transfer

The theorem of optimum power transfer is thoroughly investigated in [3] for different types of source and load impedances. Regarding students' misconceptions however, the relevant lecture started from the rather simple problem of power transfer from a DC voltage source with an ohmic internal resistance to an omhic load.

Regarding ohmic loads, a common misconception is that a large load resistance results in large power transfer from the source to the load. Students' argument is that a large load resistance results in maximum load voltage, however such a consideration ignores the fact that a large load resistance would reduce the load current that, in turn, would prevent the optimization of the power transfer. To make the aspects of the optimum power transfer more apparent, the presentation started with a simple problem which regarded the circuit of figure. 3 containing a constant voltage source v = 12V with an internal resistance $R_S = 6\Omega$ and a variable load resistance R_L with values 2, 4 , 6 , 8 and 10Ω. The students were asked to form table 1 that includes the values of the voltage, the current and the power of the load. It was easy to observe that, unlike the voltage that was increasing and the current that was decreasing, the optimum power transfer, occurred when $R_L = R_S = 6\ \Omega$ and accounted to only half of the source's power. The solution of the problem was followed by a more formal proof that handles power optimization as a maximization problem.

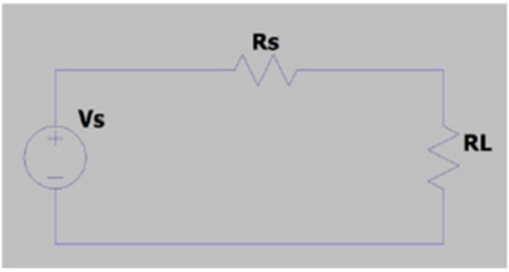

Figure 3: A single-loop circuit to demonstrate the condition $R_L = R_S$ for optimum power transfer from a voltage source (with an ohmic internal resistance) to an ohmic load.

Table 1: Voltage v_L, current i_L and power P_L at the load resistance R_L.

R_L (Ω)	v_L (V)	i_L (A)	P_L (W)
2	3	1.5	4.5
4	4.8	1.2	5.76
6	6	1	6
8	6.86	0.86	5.92
10	7.5	0.75	5.63

$v_S = 12V$, $R_S = 6\Omega$, $v_L = v_S \dfrac{R_L}{R_S + R_L}$, $i_L = \dfrac{v_S}{R_S + R_L}$ and $P_L = v_L i_L$.

A final comment (aiming to prevent students from wrongly concluding that, for a circuit at sinusoidal steady state that included complex impedances, optimum power transfer occurs when $Z_L = Z_S$) regarded the circuit of figure. 4. Though sinusoidal steady state was beyond the scope of the 1st semester course on Electric Circuits while a mathematical proof of the correct condition $Z_L = Z_S^*$ (* denoting the complex conjugate) would be tedious anyway, the following algebraic comment was made; if a = x+jy and b = u+jv (with x, y, u, v real numbers and j the imaginary unit) it is x = u whether a = b or a = b*. The point was that the condition $R_L = R_S$ for optimum power transfer between ohmic resistors does not necessarily imply that $Z_L = Z_S$ when it comes to complex impedances.

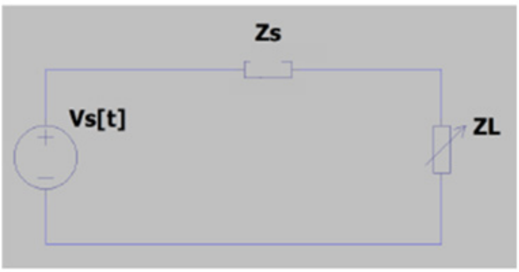

Figure 4: A single-loop circuit at sinusoidal steady state to make reference to the condition $Z_L = Z_S^*$ for optimum power transfer from a voltage source with a complex internal impedance to a complex load.

Discussion

The initiative for the described lectures was the empirical observation that several students, though capable of applying the principle of superposition, they would do it in a rather mechanical manner without appreciating the associated pre-assumptions, conditions, and limitations. Regarding the optimum power transfer, almost all students tended to relate it with the maximization of the load resistance (which results in the maximization of the load voltage) without taking into account that such an action would reduce the load current that, in turn, would prevent the optimization of the power transfer. Another misconception was that, after the students had comprehended that RL = RS is the condition for optimum power transfer between ohmic resistors, they would conclude that a similar relation ($Z_L = Z_S$) would apply for complex impedances (neglecting the fact that $Z_L = Z_S^*$ would also be compatible with the $R_L = R_S$ condition).

To enhance students' understanding and make the relevant arguments as clear as possible, a problem-based approach was applied, and students were asked to solve specifically selected problems (including counter-examples when necessary) before attending more formal proofs. The active participation of students in easy to solve problems that would, however, demonstrate the relevant arguments in an effective and comprehensible manner helped them in understanding the principles under study and acquire confidence regarding their applicability and use. A questionnaire that was given to the class after the lectures showed that the students had enhanced their understanding on the presented theorems to a very satisfactory extent.

Conclusions

The basic aim of the described lectures was the students to dissolve misconceptions and, at the same time, appreciate pre-assumptions, applicability, and limitations regarding two basic circuit theorems, the principle of superposition and the optimum power transfer. The presentation came after observations that students would apply superposition without really checking whether pre-assumptions and necessary conditions were fulfilled and, also, that they tended to connect the optimization of power transfer with the maximization of voltage load. To enhance understanding and help students grasp the relevant topics, the presentation started with a problem-solving approach (where students were actively involved) before proceeding to a brief presentation of more formal proofs. A questionnaire that was given after the lectures showed that the students had comprehended the various aspects of the presented theorems to a very satisfactory extent.

Conflict of interests: The authors declare no conflict of interests.

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