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Dealing with Mathematically Difficult Topics in Electrical and Electronic Engineering

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Abstract

Due to insufficient mathematical background, electrical and electronic engineering students may have difficulties in handling essential concepts and processes related to their field. For example, it has been observed that students find it difficult to comprehend topics such as Fourier analysis, noise, Maxwell equations and wave propagation etc. The article describes an attempt to approach such concepts in a two-step manner, first the topics are addressed in an introductory way (by means of particular, yet representative, examples and simple mathematics) and, when the students have got the necessary background and confidence, the topics are addressed in a more formal manner. As a case-study, the proposed approach has been applied to topics such as basic signal quantities, Fourier analysis, noise and chromatic dispersion in optical fibers.

keywords: Engineering education; Electrical engineering education; Telecommunications

Introduction

It has been observed that students, owing to insufficient background in mathematics, find it difficult to comprehend essential mathematical operations associated with electrical and electronic engineering subjects. This affects students' understanding of the physical and technological aspects of the above issues and makes presentation of associated phenomena a rather difficult task.

To tackle this problem, we have chosen to approach mathematically difficult topics in a two-step manner. The topics are first presented in an introductory way by using simple mathematics through representative examples and possibly computer simulations. Then, when the students have gained the necessary background and confidence, the topics are presented in a more formal manner. The main idea is that if the students get familiar and start feeling comfortable with the topics' basics, they will find it easier to approach them in a more formal way. As a case study (and in the framework of a telecommunication systems course) we have applied this approach regarding topics such as basic signal quantities, Fourier analysis, noise (as a statistical phenomenon) and signal degradation phenomena (such as the chromatic dispersion in optical fibers).

Topics to be examined

The following topics (that have been observed to cause difficulties to students) have been chosen as case-studies:

i. Basic signal quantities;
ii. Fourier analysis (Fourier series transform);
iii. Noise (as a statistical phenomenon);
iv. Chromatic dispersion in fibers (as a signal degradation process).

The above issues have been chosen based on their importance as well as on the difficulties students have in understanding and handling them. For example, Fourier analysis is an essential tool for the study of signals and their transmission through telecom networks, however students often fail to understand its actual meaning and the reasoning of its use. Noise is the most important link degradation phenomenon while dispersion is an essential limiting factor regarding the feasible bit-rate in fiber-optic links. All the above topics are considered difficult for students not only from the mathematical but also from the physical/practical point of view.

Topic 1: Basic signal quantities

It has been observed that students have difficulties in understanding the definition of the signal's average value

μ=1/T ∫▒x(t)dt▒ (1)

Here, μ is presented as the result of the operation

μ=(x(t_1)Δt_1+x(t_2)Δt_2+"..." +x(t_n)t_n)/(Δt_1+"Δ" "t" _2+"..." +"Δ" "t" _n) (2)

which is nothing more than the weighted average of a signal with a value x(t1) for a time interval Δt1, a value x(t2) for Δt2 etc (Δt1+Δt2+…+Δtn = T). For students that are already familiar with probability basics, a reference is made to the equivalence of (1) with the mean value definition where f(x) is the probability density function.

Another difficult concept is that of the impulse function δ(t). At first, this is presented as a rectangular pulse p(t) of duration τ and amplitude A= . As τ▒0, A▒ with Aτ (the surface under p(t), that is the integral) remaining equal to 1. Students are now prepared to appreciate the formal definition of δ(t) which is

δ(t) = 0 (t▒0) and

δ(t) = 0 (t▒0) and ∫_(0-)^(0+)▒δ(t)dt▒=1 (3)

Topic 2: Fourier analysis

Fourier analysis, as a mathematical tool, is encountered in many science and engineering branches. Regarding telecommunications, Fourier analysis apart from uncovering the frequency content of signals, it can greatly simplify treatment of signal transmission through systems due to, among others, its possibility to convert complicated mathematical processes (such as differential/integral equations and convolution) of the time-domain to algebraic processes in the frequency domain [1,2].

The presentation of Fourier analysis starts by focusing at the rectangular pulse-train and the rectangular pulse signals. The reasons are the following:

Rectangular pulse-train (an essential signal for digital transmission) is rather easy to express by means of a Fourier series and offers itself for illustrating the physical meaning of the process. To simplify algebra and associate the signal to actual digital transmission situations, the duty-cycle d = (τ being the pulse duration τ and T the signal period) is chosen equal to 0.25 and 0.5.

The rectangular pulse-train is suitable for illustrating the relation between the complex and the trigonometric form of Fourier series. It is also suitable for demonstrating the effect of frequency-sensitive devices (such as filters) on transmitting signals.

The rectangular pulse-train offers itself for illustrating the transition from Fourier series to Fourier transform. Indeed, by increasing the period T (practically by moving away side pulses and only leaving the central one) the pulse-train signal converts to a single rectangular pulse which is non periodic and has to be analyzed by means of Fourier transform.

The rectangular pulse-train is suitable for transition to the impulse train while the rectangular pulse can be used to demonstrate Fourier analysis of e.g. the DC signal (pulse duration τ ▒▒) and the impulse signal δ(t) (τ▒0).

Comments and discussion on the rectangular pulse-train regard, among others, the following issues:

The form and shape of Fourier coefficients (with a reference to their physical meaning);

The physical meaning of the DC coefficient and its property to be the average value of the pulse-train signal.

The physical meaning of the first zero Fourier coefficient and how it is associated to the bandwidth of the pulse train;

The power contained up to the first zero harmonic as a percentage of the total signal power and how it is related to the bandwidth we choose to assign to the pulse-train signal.

The reason that allows a digital telecom channel to have a smaller bandwidth than that of the pulse train;

The effect of either changing τ (with T remaining constant) or vice versa to the Fourier coefficients.

Part or the whole of the above discussion can be made by means of exercises and/or quizzes given to students.

The transition from the Fourier series to the Fourier transform "regime" is achieved by moving away all side pulses and only keeping the central one. This increases T to infinity (T▒▒) and practically means that the periodic pulse-train signal is converted to the non-periodic single-pulse one. At the same time, students realize that by converting the pulse-train into the single-pulse signal, the formerly discrete harmonics Xn approach each other and form a continuous spectrum X(f) which is nothing else than the Fourier transform of the rectangular pulse. From the mathematical point of view, this means that the Fourier-series becomes a Fourier integral and the Fourier coefficient Xn for the pulse-train becomes the X(f)df term of the Fourier integral.

Comments and discussion include:

The form and shape of Fourier transform of the pulse signal as compared to the form and shape of Fourier coefficients for the pulse-train;

The frequencies at which X(f) = 0 (with emphasis to the lower one fz = which is considered to be the bandwidth B of the rectangular-pulse signal).

The form of the pulse signal and its Fourier transform as τ ▒▒and τ ▒0.

As a final remark, Fourier analysis (whether series or transform) is presented as a tool that uncovers the frequency content of a signal and a simple explanation is given why knowledge of this content is necessary for the design of a communication link. It is also useful to make a short comparison with other relevant tools (e.g. Laplace transform) and explain (in a qualitative way) similarities and differences (including applicability).

The following questions and quizzes are given to students:

To comment on the frequency content of sinusoidal signals;

To demonstrate the construction of a rectangular pulse-train by Fourier coefficients;

To demonstrate the construction of a rectangular pulse by including half and the whole central lobe of the Fourier transform.

Topic 3: Noise

Noise (in its numerous forms) is the most important signal degradation phenomenon and, to a large extent, the design of telecom systems regards mitigation of noise effects [1,2]. The main difficulty regarding presentation of noise is the fact that noise, being a statistical

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phenomenon, it requires some previous knowledge of basic probability theory.

Before dealing with the probability issue, the students encounter noise in the time and frequency domain. Regarding time domain, a comment is that noise n(t), as a random signal, cannot be described by a usual function and, owing to its complete randomness, it has a zero average value. As far as the frequency domain is concerned, noise is basically a frequency insensitive signal in the sense that its power is evenly distributed across the whole frequency spectrum (white noise). Noise power is expressed in both the time and frequency domain.

Regarding statistical description, the discussion regards the Gauss-type function f(n) which gives the distribution of noise values around its zero average. Though f(n) is a probability density function, use of probability theory, apart from a mere reference, is avoided at this point.

The f(n) curve is analyzed based on the following observations:

The maximum of f(n) at n = 0 illustrates the fact that the most probable values of n(t) are around the zero mean value. In practical terms, this means that that the probability the noise to have a value around n = 0 (say between –α and +α) is larger than the probability for any other value interval equal to 2α;

The symmetry of the curve means that any value no is equally probable with value –no;

The probability (that is, the percentage of time) the values of n(t) being between –Pn and +Pn (Pn being the noise power) is always the same (about 68%). This percentage increases to about 97.5% for values between –2Pn and +2Pn;

The noise power Pn has to be compared to the signal power Ps (SNR = being the signal to noise ratio).

Topic 4: Chromatic dispersion in fibers

Chromatic dispersion (that is, the temporal spreading of signals as they propagate through the transmission medium) is due to the dependence of the medium parameters (mainly, the refractive index) on the frequency of operation [3,4]. Owing to the high bit-rates, chromatic dispersion is more evident in fiber links and results in limiting the feasible bit-rate / length product attained by the link. The difficulty in presenting chromatic dispersion is due to the fact that it requires some previous encounter with concepts related to the wave guidance process such as the propagation constant, the group velocity etc.

To overcome the above difficulties and, at the same time illustrate the fact that the optical fiber transmits the optical carrier modulated by the information signal , a short reference is made to the superposition of two oscillations of nearly equal circular frequencies ω ωm and ω+ωm (with ωm << ω) and the resultant oscillation[5].

$$y(t) = A\cos(\omega m t)\cos(\omega t) \quad (4)$$

Conclusions mainly regard:

The fact that the superposition process is equivalent with the modulation of a high-frequency carrier cos(ωt) by a lower frequency tone Acos (ωmt) which is the so-called "envelope" of y(t) and represents the information signal;

The fact that (4) may be considered as containing two oscillations, one associated with the lower-frequency envelope Acos (ωmt) and one associated with the high-frequency carrier cos(ωt)).

If, instead of the lower-frequency tone, a pulse p(t) is transmitted, then p(t) is the envelope depicts p(t) and the overall signal is of the form

$$s(t) = p(t)\cos(\omega t) \quad (5)$$

At this point, signal transmission e.g. through an optical fiber, is presented in the following manner:

Let a signal of the form (5) is launched at the fiber input;

If the carrier oscillator (in this case, a laser) can be considered as monochromatic (that is, as producing a single frequency ω), then the envelope p(t) is transmitted without any degradation.

However, since actual lasers rather than being monochromatic, they produce a frequency band ω Δω, instead) the pulse signal p(t) modulates multiple carriers within the laser band. Thus, if for the sake of simplicity, one considers that the laser emits two frequencies ω1 and ω2, then two component signals are produced

$$s1(t) = p(t)\cos(\omega 1t), \quad (6)$$

$$s2(t) = p(t)\cos(\omega 2t)$$

If s1(t) and s2(t) propagate either in vacuum (approximately, in air) or in a medium (such as a fiber core) whose refractive index n is independent from frequency ω, they travel at the same speed and the resultant signal s(t) = s1(t) + s2(t) retains its shape. If, however, the refractive index n depends on the frequency ω (that is, if n = n(ω)) n, then s1(t) and s2(t) travel at slightly different speeds which means that, after some time T, s1(t) and s2(t) cover slightly different distances thus s(t) gradually spreads in time. This is the so-called "chromatic dispersion" which limits the maximum bit-rate (actually, the maximum bit-rate/distance product) a fiber can achieve. Indeed, if the acceptable pulse spreading is set to D = (where is the bit duration and R the bit-rate) the feasible bit-rate R is equal to

$$R=1/\tau=1/4D \quad (7)$$

At this point, a quiz could be the students to calculate the feasible bit-rate for a link of length L (in km) which is feeded by a laser of bandwidth Δω (equivalently of linewidth Δλ in nm) and presents relative delay between s1(t) and s2(t) equal to σ (in ps/nm.km).

Though a detailed analysis of dispersion is beyond the scope of the "Communication Principles" course (this topic is examined in the subsequent "Optical Communications" course), the students are now prepared to appreciate the formal definition of the chromatic dispersion coefficient which is given by

$$\sigma=d/d\lambda\ (1/v\_g\ ) \quad (8)$$

where vg = is the group velocity (Agrawal, 2001; Agrawal 2010).

Discussion and Conclusion

The article proposes a way to address topics that, mainly due to insufficient mathematical background, the students often find difficult to approach. At the first step, an introduction is made by means of specific, yet representative examples that are presented and analyzed in detail in a way that enables more general conclusions to be obtained. Then, when the students have got a good grasp of basics and have developed the necessary confidence on the topic under study, a more formal presentation takes place.

As a case study, the described approach has been applied (within the context of a telecommunication systems course) to the presentation of basic signal quantities, Fourier analysis, noise and chromatic dispersion that the students find difficult to comprehend. Other telecommunication topics where this approach could be used are the sampling process, the digital modulation techniques (such as

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the phase shift keying – PSK), the effect of noise on the bit-error rate (BER) etc. Another group of subjects could be Maxwell equations and electromagnetic waves as well as propagation in metallic and dielectric waveguides.

An issue regarding the described approach is the increased time needed for its implementation. To save time and, at the same time, enhance active participation on behalf of the students, computer simulation could be used to illustrate synthesis of rectangular pulse trains and pulses by means of Fourier sums and integrals, the effect of changing signal parameters on the Fourier series and transform, etc.

Though a quantitative evaluation of learning outcomes is not easy to perform, the efficiency of the described approach was evaluated by means of the exam results on the topics presented that were substantially better than before. Distributed questionnaires also showed a strong preference for the described approach among the participating students.

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